WAVE REFLECTION AND WAVE RUN-UP AT RUBBLE MOUND BREAKWATERS

Markus Muttray[1], Hocine Oumeraci[2], Erik ten Oever[1]

Wave reflection and wave run-up at rubble mound breakwaters with steep front slope were investigated in large scale model tests. The two-way dependency of wave run-up and wave reflection and the governing hydraulic parameters for wave reflection were investigated. The wave run -up height is closely related to the clapotis height in front of the breakwater. An empirical wave run-up formula that includes the reflection coefficient was developed. The wave height has little influence on the wave reflection from porous structures. An empirical wave reflection formula is proposed for rubble mound structures with steep front slope.

INTRODUCTION

The wave reflection coefficient is a bulk parameter for the hydraulic processes at a breakwater or coastal structure, i.e. for wave breaking, wave penetration into the structure, wave transmission and wave overtopping. Reflection analysis is mostly performed in order to determine the incident wave conditions in front of the structure. The reflection coefficient is not further used for the interpretation of the hydraulic processes at the structure.

The main objective of this paper is (i) investigating the two-way dependency of wave run-up and wave reflection and (ii) determining the governing hydraulic parameters for wave reflection.

This study focuses on rubble mound structures with a steep front face (steeper than 1:2) as these structures are especially in deeper water more economical than gently sloping structures. Wave reflection and wave run-up were investigated in hydraulic model tests; experimental results are presented in this paper.

EXPERIMENTAL STUDY

Wave reflection and wave run-up were investigated in large scale model tests in the Large Wave Flume (GWK) in Hanover, Germany. A rubble mound breakwater with typical cross section and 1:1.5 slopes was installed; the layout of the breakwater and the measuring devices are shown in Figure 1. The breakwater had an armour layer of Accropodes (unit size 40 kg) and a core of gravel (average grain size 3.1 cm, porosity 39%). Wave run-up was measured on the armour layer; wave reflection was determined by the 3-gauge method

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(Mansard & Funke, 1981). The range of tested wave conditions comprised wave steepness H/L = 0.005 to 0.053, relative water depth h/L = 0.05 to 0.23, relative wave height H/h = 0.08 to 0.40 and breaker index $\xi \geq 6$. Details of experimental set-up, measuring devices, wave conditions and analysis can be found in Muttray (2000) and in Muttray & Oumeraci (2005).

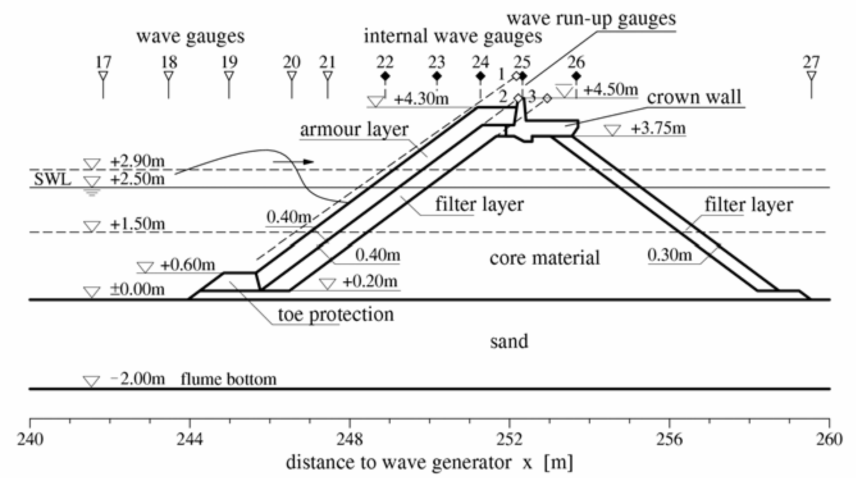

Figure 1: Cross section of breakwater model

WAVE RUN-UP

The wave reflection at the seaward face of a rubble mound breakwater causes a partial standing wave field in front of the breakwater. The water surface envelope on the foreshore and at the structure is plotted in Figure 2 for regular wave conditions.

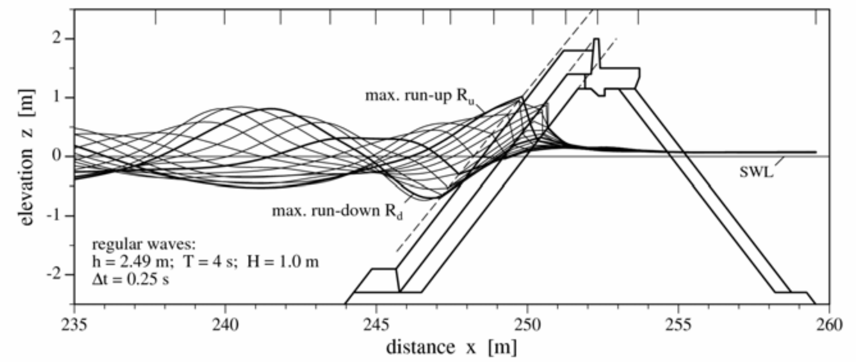

Figure 2: Water conditions in the near field and wave motion on the breakwater slope

The water surface envelope forms a knot at the breakwater toe and an anti-knot further seaward. The wave run-up and run-down on the breakwater slope can be interpreted as a slightly distorted anti-knot of the partial standing wave system. One can conclude that clapotis height and wave run-up height are probably related. The clapotis height H_c signifies the local wave height at an anti-knot:

$$H_c = H_i + H_r = H_i\,(1 + C_r) \tag{1}$$

with incident wave height H_i , reflected wave height H_r and reflection coefficient $C_r = H_r/H_i$.

The run-up height R describes the vertical distance between highest run-up level R_u and deepest run-down R_d (which is similar to the definition of wave height). The wave run-up height R is plotted in Figure 3 against the clapotis height H_c for regular and irregular waves. The run-up height for regular waves exceeds 2 m; the significant run-up height R_{m0} for irregular waves (derived from 0^{th} moment of wave run-up spectrum) reaches almost 2 m. Run-up height and clapotis height are closely related; the relation can be approximated by the following empirical equation:

$$R = a\ H_i\,(1 + C_r) \tag{2}$$

where coefficient a was found to be about 1.31 for regular waves and 1.17 for irregular waves.

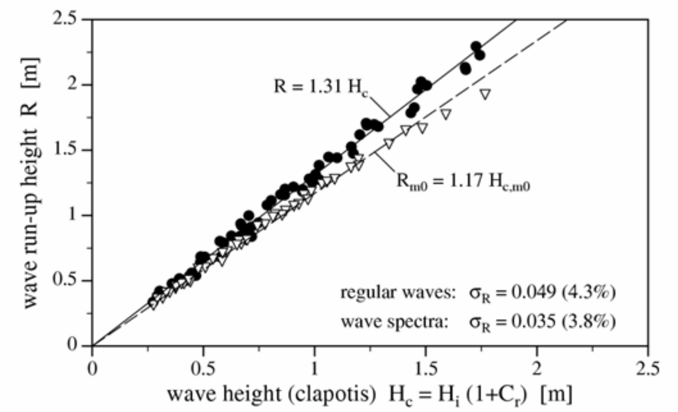

Figure 3: Wave run-up height vs. clapotis height for regular and irregular waves

The maximum wave run-up level R_u on the slope is of more practical importance than the wave run-up height R. The highest wave run-up depends on wave run-up height R and on the asymmetry of the wave run-up R_u/R. The latter was determined from experimental data.

The wave asymmetry of progressive waves (i.e. ratio of crest level and wave height η_{max}/H) can be determined by higher order wave theory. The wave asymmetry for uniform waves on a horizontal seabed can be approximated according to Muttray (2000) by:

$$\frac{\eta_{\max}}{H} = \frac{1}{2} + \frac{1}{3}\frac{\Pi}{\Pi + 1/2} + \frac{1}{6}\frac{\Pi^2}{\Pi^2 + 1/30}$$
$$\Pi = \frac{H}{L}\coth^3\left(\frac{2\pi}{L}h\right) = \frac{H}{L}\left(\frac{L_0}{L}\right)^3 \tag{3}$$

The approximation according to equation 3 is based on Fourier wave theory and confirmed by experimental results (Muttray, 2000).

The wave asymmetry of partial standing waves was determined experimentally at the first anti-knot in front of the breakwater. The results are plotted in Figure 4. The asymmetry of a partial clapotis is about 2/3 of the asymmetry of a progressive wave and can be approximated by:

$$\frac{\eta_{\max}}{H} = \frac{1}{2} + \frac{2}{9}\frac{\Pi}{\Pi + 1/2} + \frac{1}{9}\frac{\Pi^2}{\Pi^2 + 1/30} \tag{4}$$

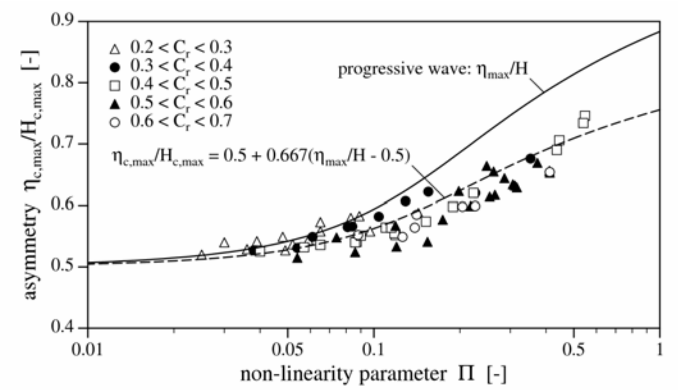

Figure 4: Wave asymmetry of a partial standing wave system (at anti-knot)

The asymmetry of the wave run-up is plotted in Figure 5. The run-up asymmetry is almost identical to the asymmetry of a partial clapotis and can be approximated by equation 4 (replacing η_{max}/H by R_u/R).

Irregular wave tests were performed with TMA wave spectra. Non-breaking wave conditions were investigated, i.e. no wave breaking occurred on the foreshore. The distribution of incident wave heights and wave run-up heights is almost identical. In this case they are both Rayleigh distributed. The maximum run-up height for irregular waves (or a run-up height with a specific probability of exceedence) can be derived from equation 4 (replacing η_{max}/H by R_u/R) and 2. The significant wave height shall be replaced in equation 2 by the maximum wave height (or by the wave height with the corresponding probability of exceedence). Typical run-up heights are as follows: R_{max} = 1.87 R_{m0} (based on 1000 waves); $R_{2\%}$ = 1.41 R_{m0}; $R_{10\%}$ = 1.27 R_{m0} and $R_{50\%}$ = 0.63 R_{m0}.

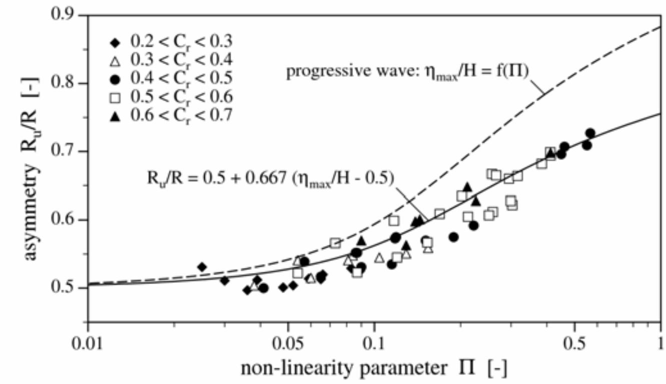

Figure 5: Wave asymmetry partial standing wave system

WAVE REFLECTION

In hydraulic model studies wave reflection is typically determined from the spatial variation of wave conditions. Wave gauge arrays are used for this analysis. Wave reflection cannot be directly measured; the uncertainties of the reflection coefficient are thus significantly larger than the uncertainties of directly measurable wave parameters like local wave height or wave pressure.

The wave reflection at a non-overtopped rubble mound structure is determined by two processes:

- Wave energy dissipation on the slope, which is mostly wave breaking;
- Wave penetration into the structure.

The wave reflection from impermeable slopes can be assessed according to Muttray & Oumeraci (2002) by:

$$
\begin{aligned}
C_r &= 1-\left(\frac{H_0}{H_{0,crit}}\right)^{3/2}\left(1-\frac{2}{\pi}\right) && ;\ \frac{H_0}{H_{0,crit}} < 1 \ \text{(non-breaking waves)} \\
C_r &= \frac{2}{\pi}\frac{H_{0,crit}}{H_0} && ;\ \frac{H_0}{H_{0,crit}} \geq 1 \ \text{(breaking waves)} \\
H_{0,crit} &= L_0\sqrt{\frac{2\alpha}{\pi}\frac{\sin^2\alpha}{\pi}}
\end{aligned}
\tag{5}
$$

The terms 'non-breaking' and 'breaking' refer to wave breaking on the breakwater slope (with slope angle α between slope and horizontal) and not to depth-induced wave breaking on the foreshore. This approach is based on Miche's reflection hypothesis (Miche, 1951). When the actual wave steepness H_0/L_0 exceeds the critical wave steepness $(H_0/L_0)_{crit}$ waves start breaking; the incident wave energy is partly dissipated (by wave breaking) and partly reflected. The critical deep water wave steepness $(H_0/L_0)_{crit}$ was derived by Miche (1951) theoretically from second order Stokes wave theory. Wave reflection is thus proportional to $H_{0,crit}/H_0$; the reflection coefficient increases with decreasing wave steepness (i.e. $C_r \propto L/H$).

The following empirical approach has been proposed by Muttray & Oumeraci (2002) for the wave reflection from porous walls:

$$C_r = 1 - [1-(1-n)^3]^{1/6} + (1-n^{5/6})\,\frac{H_i}{h} \tag{6}$$

This approach is applicable for vertical walls with an opening ration n of 12% to 40%. The wave reflection is not affected by the wave length. The reflection coefficient increases with decreasing opening ratio n and with increasing relative wave height H_i/h. (i.e. $C_r \propto n^{-1}, H/h$).

The wave reflection from a rubble mound structure will probably vary with wave steepness and with relative wave height. The effect of wave height, wave length and water depth on the wave reflection from a rubble mound breakwater was determined from experimental data (see Figure 6). It can be seen that:

- The wave reflection is primarily affected by the wave period; the reflection coefficient is proportional to the wave period ($C_r \propto T$);
- The wave reflection is slightly decreasing with water depth ($C_r \propto h^{-1}$);
- The effect of wave height on wave reflection is negligible.

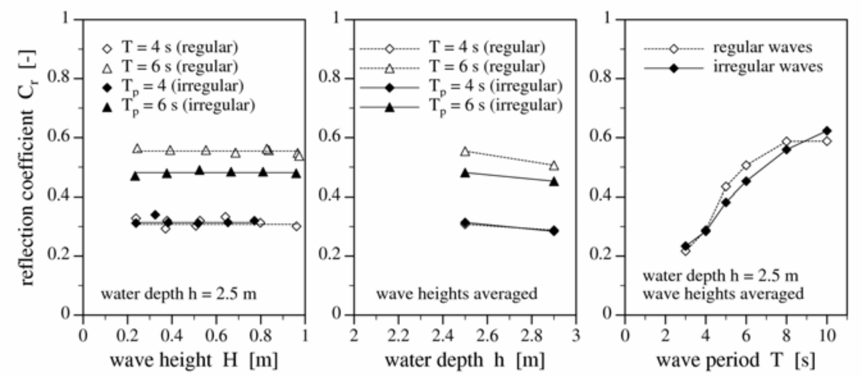

Figure 6: Reflection coefficient vs. wave height, water depth and wave period

The effect of wave breaking (i.e. decreasing reflection coefficient with increasing wave height) and the effect of permeability (i.e. increasing reflection coefficient with increasing wave height) are approximately balanced. The effect of wave height on the reflection coefficient is almost evened out.

A linear wave reflection approach (i.e. an approach that is independent of wave height) has been derived. It is assumed that $C_r \propto T^2/h$ (or $C_r \propto L_0/h$). The wave reflection from rubble mound structures with steep front face can be approximated by the following empirical approach:

$$C_r = \frac{1}{1.3 + 3\,h\,\dfrac{2\pi}{L_0}} \qquad (7)$$

Measured reflection coefficients (from regular and irregular wave tests) and predicted reflection coefficients according to equation 7 are plotted in Figure 7. The standard deviation between measured and predicted reflection coefficients is 0.06 (14%) for regular waves and 0.02 (5%) for irregular waves.

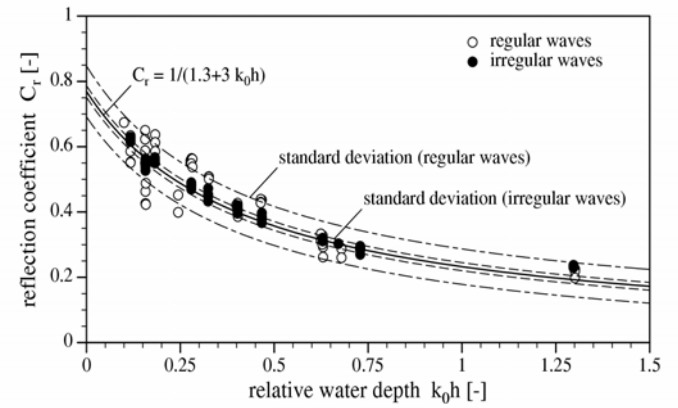

Figure 7: Reflection coefficients for regular and irregular waves

The new reflection approach is applicable for regular and irregular waves. As this approach is independent of wave height (linear approach) it can be also applied as a frequency dependent transfer function between incident and reflected wave spectrum.

Most literature approaches use the ratio of slope angle and wave steepness as governing parameter for the wave reflection. These approaches are apparently focused on the hydraulic processes on the slope (i.e. wave breaking) and are mostly neglecting the effect of porosity (i.e. wave penetration). Hence, the wave penetration into the structure would be according to these literature approaches a secondary effect.

Empirical wave reflection approaches that use the breaker index $\xi = \tan\alpha / \sqrt{H/L}$ as governing parameter are a.o. Battjes (1974), Gimenez-Curto (1979), Seelig & Ahrens (1981), Buerger et al. (1988), Postmar (1989), Davidson et al. (1996) and Zanuttigh & van der Meer (2006). A modified breaker index is applied a.o. by van der Meer (1992) and Hughes & Fowler (1995). They assume implicitly that (i) wave energy dissipation on the breakwater slope is determined by wave breaking and (ii) wave energy that is not dissipated will be reflected (reflection hypothesis of Miche, 1951). Predicted reflection coefficients according to above empirical formulae were compared with the experimental results from irregular wave tests; the outcome is summarised in Table 1.

Author	Breaker index ξ	Reflection formula C_r	Mean error[1)]	Standard deviation abs.	Standard deviation rel.
Battjes (1974) [2)]	$\frac{\tan\alpha}{\sqrt{H/L_0}}$	$0.1\xi^2$	2.34	0.59	138%
Gimenez-Curto (1979)	$\frac{\tan\alpha}{\sqrt{H/L_0}}$	$\frac{1}{2}-\frac{\exp(-0.125\,\xi)}{2}$	0.67	0.16	37%
Seelig & Ahrens (1981)	$\frac{\tan\alpha}{\sqrt{H/L_0}}$	$\frac{0.6\,\xi^2}{6.6+\xi^2}$	1.20	0.12	28%
Buerger et al. (1988)	$\frac{\tan\alpha}{\sqrt{H/L}}$	$\frac{0.6\,\xi^2}{12+\xi^2}$	1.09	0.10	24%
Postmar (1989)	$\frac{\tan\alpha}{\sqrt{H/L_0}}$	$0.125\,\xi^{0.73}$	1.23	0.14	33%
van der Meer (1992) [3)]	$\frac{\tan^{0.62}\alpha}{(H/L_0)^{0.46}}$	$0.07\,(P^{-0.08}+\xi)$	1.23	0.16	37%
Hughes & Fowler (1995)	$\sqrt{\frac{h}{gT^2\tan\alpha}}$	$\frac{1}{1+7.1\,\xi^{0.8}}$	0.94	0.06	14%
Davidson et al. (1996) [4)]	$\frac{\tan\alpha}{\sqrt{H/L_0}}$	$0.298\ln\xi + f\left(D,P,h,\alpha,H,L_0\right)$	0.88	0.09	20%
Zanuttigh & van der Meer (2006)	$\frac{\tan\alpha}{\sqrt{H/L_0}}$	$\tanh(0.12\,\xi^{0.87})$	1.32	0.15	35%
This study	–	Equation 7	0.99	0.02	5%

[1)]Mean error $=\frac{1}{n}\sum_{i=1}^{n}\frac{C_{r,calculated}}{C_{r,measured}}$

[2)]Not applicable for rubble mound structures

[3)]Includes besides breaker index wave transmission D/H, permeability P, roughness of slope ($\sqrt{D/L_0}\cot\alpha$) and relative water depth at the toe h/L_0 (with rock diameter *D*)

[4)]Permeability coefficient $P=0.4$ for multi layered rubble mound structures

[5)]Uses wave period $T_{m-1,0}$ (= m_{-1}/m_0) instead of peak wave period

Table 1: Applicability of empirical wave reflection formulae for rubble mound structures with steep front face

The predicted reflection coefficients deviate significantly from measured coefficients. The approach of Hughes & Fowler (1995), which does not include the wave height, provides the best approximation with a relative standard deviation of 14%. The relative standard deviation of all other approaches that use the wave steepness as governing parameter exceeds 20%.

Little variation of the breaker type can be observed on slopes steeper than 1:2 according to Muttray & Oumeraci (2002). The experimental results indicate that the effect of wave breaking and the effect of permeability are almost balanced (see Figure 6). The reflection coefficient is thus almost independent of wave height. Predictive equations based on breaker index ξ overestimate the effect of wave breaking. They are apparently not applicable for rubble mound breakwaters with steep front slope as they do not include the effect of permeability.

CONCLUSIONS

The wave run-up is closely related to clapotis height in front of the structure. A linear relation was found between run-up height R and clapotis height H_c at the breakwater toe. The wave reflection is the key to a simple deterministic wave run-up model. The reflection coefficient has been applied for the wave run-up prediction and will be probably also applicable for a deterministic wave overtopping model. An empirical wave run-up formula, which includes the reflection coefficient, has been developed (equation 2).

Predictive equations for wave reflection that are based on the breaker index ξ overestimate the effect of wave breaking. They are not applicable for rubble mound breakwaters with steep front slope. An empirical wave reflection formula is proposed that is based on the relative water depth h/L_0 (equation 4).

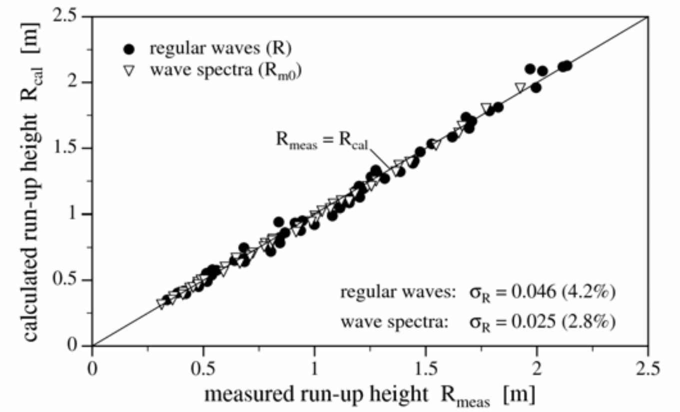

Figure 8: Measured vs. predicted wave run-up for regular and irregular waves

Reflection and run-up formulae are applicable for regular and irregular waves. The reflection formula can be also applied as a transfer function between incident and reflected wave spectrum. The formulae shall be applied only for non-breaking waves (i.e. no wave breaking on the foreshore) and for conditions with little or no wave overtopping. The wave run-up formula with empirical coefficients a = 1.2 – 1.3 is only applicable for 1:1.5 slopes. The reflection formula is applicable for rubble mound structures with a porosity of about 40% and with 1:1.5 slopes.

A comparison of measured and predicted wave run-up heights (according to equations 2 and 4) is plotted in Figure 8. Measured and calculated wave run-up are in close agreement, the standard deviation is less than 5 cm (4%) for regular waves and 3 cm (3%) for irregular waves.

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Abstract 713

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